The effect of the choice of the control variables of the water level control of open channels

Comparison of control schemes using the ASCE Test canal 2

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Abstract— *The effect of the choice of the control action variables on centralized water level controllers for open channels is analyzed. Three models are compared. In the first model the control action variable is the discharge and then the inverse gate equation is used to calculate the gate openings. In the second case the control action variable is the gate opening and that is incorporated to the canal model – supposing that the upstream water levels of each pool are known. In the third case control variables are also the gate openings but the upstream water level of each canal pool is unknown, they are calculated by the models by using the hydraulic relationships between the variables. These three models are discussed and compared through an example of centralized Linear Quadratic Regulator (LQR) controller using as example the Test canal 2 of the ASCE.*

Keywords —*automatic control, irrigation, canal, integrator, delay, gate*

I. INTRODUCTION

An irrigation canal with several reaches is a complex system where each reach can be considered as a subsystem. These subsystems are coupled through the discharge under the gates. A change in the opening of one gate affects the gate discharge of the gates upstream and downstream of the given gate and also the water levels upstream and downstream. This new change in the discharge can be considered as a perturbation that travels upstream and downstream. This effect is stronger in flat canals, with low friction, but it is present in any canal under subcritical flow.

In order to develop a distant downstream controller (either water level or discharge is controlled) the choice of the control action variables can be the upstream discharge or the upstream gate opening. Both of these approaches are commonly used in canal control [1]. The difference between the two approaches is discussed below, first in the case of decentralized control and second in the case of centralized control.

In case of decentralized control, several controllers are trying to control individual systems that are in fact heavily coupled. For two canal reaches connected by a gate, the gate opening can be the control action variable for the downstream reach, while it is an unknown perturbation for the upstream reach. Not taking this effects into account can lead to disturbance amplification [2] and unacceptable controller performance. One way to decouple these variables is using discharge as control action variable instead of gate opening. In this case the gate opening is set by a slave controller, taking into account the water level upstream of the gate that belongs to the other canal pool. The slave controller can have several configurations, the most simple is the inverted gate equation. A better approximation is to take into account the change in water levels by using the method of characteristics [3] or the integrator delay zero model [4]. In [3] different possible configurations with different canal geometries are compared by using PI controllers. The best results were achieved by using discharge as control action variable and a slave controller that takes into account the water level changes.

For centralized systems no such tests have been carried out. In case of using discharge as control action variable the internal model has no direct information about the effect of the change of the water levels caused by the change of the opening. Moreover, the controller has no information about the change of discharge further propagated upstream. This information enters the controller when they occur in nature (after a certain delay) and then the controller is able to react to that. Hence, these type of controllers as first action can only act on the gates that are the neighbors of the gate where the opening occurred. In order to develop a controller that is aware of this dynamics and can act faster, the dynamics of the gates should be considered and implemented [1]. The gates in this case need to be modeled. It is possible to be carried out by identification experiments or by linearizing the gate equation. In both cases the problem is if the model is used in a regime far from the one where it was linearized. This problem can be overcome by using multiple models.

In this work the choice of the influence of the control action variables on centralized controllers is investigated, whether discharge or gate opening is more advantageous. There are different models shown, two of them including the linearized gate dynamics in the overall state space models. The models are introduced in Sections II and III. In Section IV, the models are used to design and compare LQR controllers, discussing the results in Section V.

II. GATE MODELING

In order to combine the model of gates into the state space model of the system the classical gate equation is linearized. The equation is the following:

$$Q = C_d L b \sqrt{2g\left(H_1 - H_2\right)}, \tag{1}$$

where Q is the discharge under the gate, C_d is the discharge coefficient, L is the gate opening, b is the width of the gate, g is the acceleration of gravity, H_1 is the water level upstream and H_2 is the water level downstream. It can be linearized around the steady state Q_0, L_0, H_{10}, H_{20}, The deviations from this steady state are noted by q, l, h_1, h_2. For example, the level deviation is

$$h_1 = H_1 - H_{10}. \tag{2}$$

Hence Eq. 1 becomes

$$q = l k_l + h_1 k_{h1} + h_2 k_{h2}. \tag{3}$$

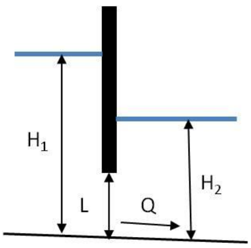

Fig. 1. Schematic drawing of a canal with the water levels used for gate modeling

The coefficients in the linearized gate equation can be obtained as the partial derivatives by the given variable:

$$k_{h1} = \frac{\partial Q}{\partial H_1} = \frac{1}{2} C_d L_0 b \sqrt{2g} \frac{1}{\sqrt{\left(H_1 - H_2\right)}} \tag{4}$$

$$k_{h2} = \frac{\partial Q}{\partial H_2} = -\frac{1}{2} C_d L_0 b \sqrt{2g} \frac{1}{\sqrt{\left(H_1 - H_2\right)}} \tag{5}$$

$$k_l = \frac{\partial Q}{\partial L} = C_d b \sqrt{2g \;\; H_1 - H_2}. \tag{6}$$

These gains are considered to be constant within a certain range around the values where the equation was linearized.

III. THE MODEL OF THE CANAL

A Linear Quadratic Regulator (LQR) controller has been developed. The structure of the state space model is described in details in [5]. Here we present only the basic hydraulic equations used to build the overall model.

The canal was modeled using the Integrator Delay (ID) model [2] whose parameters have been calculated using the physical characteristics of the canal. This model provides the relationship between the water level and the discharge in the following form:

$$h_{1i}\left(k+1\right) = h_{1i}\left(k\right) + A_{di} q_i \left(k - d_{1i}\right) - A_{di} q_{i+1}\left(k\right) \tag{7}$$

$$h_{2i}\left(k+1\right) = h_{2i}\left(k\right) + A_{di} q_i \left(k\right) - A_{di} q_{i+1}\left(k - d_{2i}\right) \tag{8}$$

where h_i is the water level (relative to the point of linearization), A_{di} is the discretized backwater area, d_{1i} is the delay between a change in the upstream discharge and the downstream water level and d_{2i} is the delay between a change in the downstream discharge and the upstream water level. The index i indicates the number of the reach.

In this work three different models are considered as following below.

A. Model 1

The first model is the same as it was used in [5]. This model considers the change in discharge q_i as control variable for every canal reach. The state space is built using the following equations:

$$\begin{cases} h_{1i} \;\; k+1 \;\; = h_{1i} \;\; k \;\; + A_{di} q_i \;\; k - d_{1i} \;\; - A_{di} q_{i+1} \;\; k \\ q_i \;\; k+1 \;\; = q_i \;\; k \;\; + \Delta q_i \;\; k+1 \\ h_{\text{int}\,i} \;\; k+1 \;\; = h_{\text{int}\,i} \;\; k \;\; + h_{1i} \;\; k \end{cases} \tag{9}$$

where h_{int} is an integral variable that was introduced in order to eliminate the steady state error in the control operations.

In this case, the gate openings are calculated a posteriori by measuring the water level downstream of the gate and the gate equation is simply reversed. Using this state space representation, a change in the discharge in a canal pool causes a change only in the water level of the same pool and in the water level of the canal pool upstream to it. It does not cause any change in the discharges in the state, since all the discharges are influenced only by the control variable (that is the change in discharge). A simple test is carried out using a canal of 8 canal pools, with constant water level in the reservoir upstream and constant downstream discharge. The canal pools are connected by sluice gates. In Fig. 2, the discharge under Gate 5 is increased and the response of the water levels can be seen. The water level in the Pool 4 (directly upstream of Gate 5) decreases and the water level in Pool 5 (directly downstream of the Gate 5) increases. The disturbance does not travel upstream or downstream in the canal according to this model, while in reality it does as it can be seen from the numerical solution of the Saint-Venant equations.

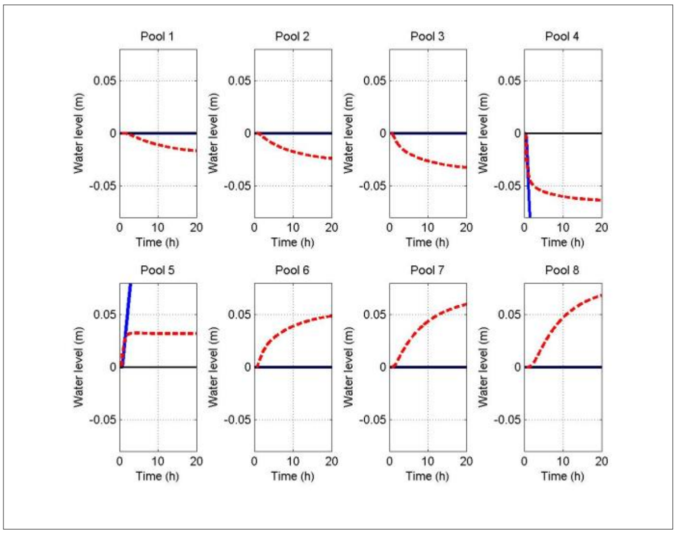

Fig. 2. Response of Model 1 to increasing of the discharge at Gate 5, with dotted line the response of the hydrodynamic model and with straight line Model 1

B. Model 2

In case of Model 2, the linearized gate Eq. 1 for each canal pool are used to build the state space, and the water levels downstream of the gate (h_2) are like "measurable perturbations". In this case the model allows the perturbations (changes in water level) propagate downstream, hence the model has knowledge about this action.

The state space is built using the following equations:

$$\begin{cases} h_{1i}(k+1) = h_{1i}(k) + A_{di}q_i(k-d_{1i}) - A_{di}q_{i+1}(k) \\ q_i(k+1) = l_i(k+1)\,k_{li} + k_{h1i}h_{i-1}(k+1) + k_{h2i}h_{2i}(k+1) \\ h_{\text{int}\,i}(k+1) = h_{\text{int}\,i}(k) + h_{1i}(k) \\ l_i(k+1) = l_i(k) + \Delta l_i(k+1) \end{cases} \tag{10}$$

where q_i is the discharge in the i^{th} pool, l_i is the gate opening of the i^{th} gate, k_{hi} is the gain of the i^{th} gate on the water level upstream of the gate (at the downstream end of the previous canal pool) and k_{hui} is the gate of the water level downstream of the gate (the upstream end of the canal pool).

In this model, the control action variable is the change in the gate opening. The water levels (h_2) at the upstream part of the gates are considered as measured variables. Therefore they are part of the state, and are updated every time from measured data.

The same experiment was carried out that in case of Model 1, but instead of increasing the discharge at Gate 5, the gate opening was increased. Fig. 3 shows the results in water level. During this experiment the water level downstream of the gate was not measured, therefore it is considered to be constant for the model. It can be seen that the existence of the perturbations downstream that happen in reality are predicted by the model, however, the magnitude and the final state is different from the real values due to the fact that the water levels upstream are not taken into account in this model, which at this experiment are considered constant. It can also be noted that, while in reality (the results of the numerical solution of the Saint-Venant equations) the perturbations also travel upstream, this is not predicted by Model 2.

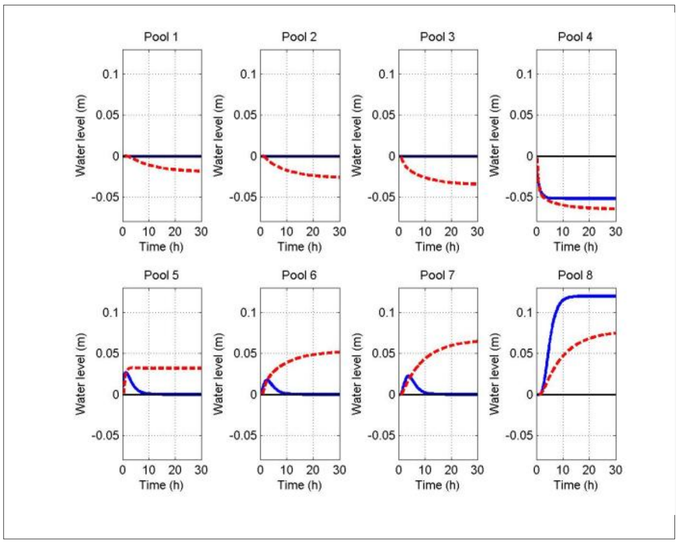

Fig. 3. Response of Model 2 to increasing the opening of Gate 5, with dotted line the response of the hydrodynamic model and with straight line Model 2

C. Model 3

In case of the third model, the linearized gate equation is still used as in Model 2. However, in this case, the water levels (h_2) at the upstream part of the gates are not measured variables. Instead, they are related to discharges by using models as the one in Eq. 8.

The state space of Model 3 uses the following set of equations:

$$\begin{cases} h_{1i}(k+1) = h_{1i}(k) + A_{di}q_i(k-d_{1i}) - A_{di}q_{i+1}(k) \\ q_i(k+1) = l_i(k+1)\,k_{li} + k_{h1i}h_{i-1}(k+1) + k_{h2i}h_{2i}(k+1) \\ h_{2i}(k+1) = h_{2i}(k) + A_{di}q_i(k) - A_{di}q_{i+1}(k-d_{2i}) \\ h_{\text{int}\,i}(k+1) = h_{\text{int}\,i}(k) + h_{1i}(k) \\ l_i(k+1) = l_i(k) + \Delta l_i(k+1) \end{cases} \tag{11}$$

The result of this combined model is that the effects of water level changes can propagate downstream and upstream as well.

This can be seen in Fig. 4. The same experiment is carried out as in case of Model 2: the opening of Gate 5 was increased. The advantage of this model is twofold: (1) it is able to reproduce the disturbances travelling in both directions, and (2) it does not need measured data about the water levels at the upstream end of the canal pools.

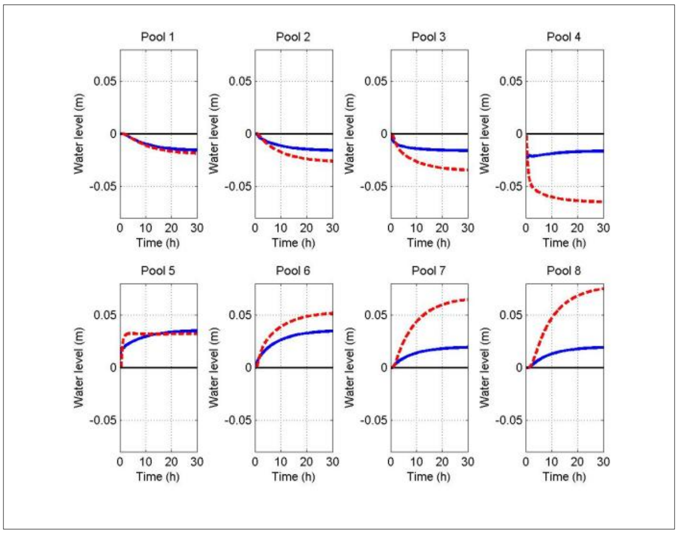

Fig. 4. Response of Model 3 to increasing the opening of Gate 5, with dotted line the response of the hydrodynamic model and with straight line Model 2

IV. Controller Development and Test

A. LQR controller

All the three models were transformed into state space form:

$$x(k+1) = Ax(k) + Bu(k) + B_d D(k) \qquad (12)$$

where x is the state vector, u is the input control vector, D is the disturbance vector, and A, B, and B_d, are matrices of appropriate dimensions. In brackets, k is the present time and $k+1$ is the future time step.

In all the three cases a discretization time of 500 s was used. The delay steps are calculated for each reach as the maximum of the upstream and downstream delay steps in order to be able to represent the downstream and upstream disturbance propagation.

In order to keep the process as close as possible to a predefined reference, the optimal control process was carried out using the following objective function:

$$\min_{\Delta u} J = \sum_{j=0}^{\infty} e_{1i}^T(k+j|k)\, Q_e e_{1i}(k+j|k) + \sum_{j=0}^{\infty} e_{\mathrm{int}\,i}^T(k+j|k)\, Q_{\mathrm{int}} e_{\mathrm{int}\,i}(k+j|k) + \sum_{j=0}^{\infty} \Delta u^T(k+j|k)\, R \Delta u(k+j|k), \qquad (13)$$

where e is a vector containing the water level errors for the three pools for the whole prediction horizon, Q_e is the weighing matrix for the error, e_{int} is a vector containing the integral of the errors for the whole prediction horizon, Q_{int} is the weighing matrix for the integral of the errors, u is a vector containing the inputs (change in discharge or change in gate opening) and R is the weighing matrix for the input.

B. Tuning

In case of this LQR controller there are three parameters to tune: the penalty on the water level error, the penalty on the integral (sum) of the water level error and the penalty on the control action variable (discharge in the first case and gate opening in the second case). In order to make a fair comparison between the three controllers, similar conditions should be used. Therefore the penalties on the error and integral error were chosen to be the same. The penalty on the control action variable should be different depending on its type: discharge or gate opening. Therefore the following process was established: an objective function was created based on the weighted sum of the performance indices, and the penalty on the control action variable was changed in order to minimize this objective function. As it was expected, in the two cases where the control action variable is the gate opening the same penalty minimized the function. The penalties were reciprocals of the square of the absolute affordable error (10 cm and 50 cm for the integral variable) and change in the control action variable (0.15 m^3/s in case of discharge and 0.05 m in case of gate opening).

C. The test canal

The controllers are tested on the Corning canal, the second test canal of the ASCE [6]. In order to compare these controllers, the first half of the Test-2-1 was carried out in tuned conditions. The test lasts 24 hours and the objective is to keep the water levels constant while disturbances occur. At the downstream end of each pool there is a gravity offtake. At 2h, the gates at offtakes 5 and 6 are opened to increase the offtake discharge from 1 m^3/s to 1.5 m^3/s and 2 m^3/s respectively.

The performance of the controllers is analyzed by using the performance indices suggested by the ASCE [6], which are three indicators computed for each controlled variable (downstream water level): the maximum absolute error (MAE), the integral of the absolute magnitude of the error (IAE), the steady state error (StE), the integrated absolute discharge change (IAQ) and the integrated absolute gate movement (IAW).

Table 1. The results of Model 1

	MAE (%)	IAE (%)	StE (%)	IAQ (m^3/s)	IAW (m)
Max	9.66	3.15	2.10	6.30	0.95
Avg	4.58	1.47	0.95	2.94	0.50

Table 2. Results of Model 2

	MAE (%)	IAE (%)	StE (%)	IAQ (m^3/s)	IAW (m)
Max	7.52	2.15	0.37	8.19	0.51
Avg	3.71	0.97	0.18	2.41	0.28

Table 3. Results of Model 3

	MAE (%)	IAE (%)	StE (%)	IAQ (m^3/s)	IAW (m)
Max	7.51	2.15	0.37	8.23	0.52
Avg	3.71	0.98	0.18	2.43	0.28

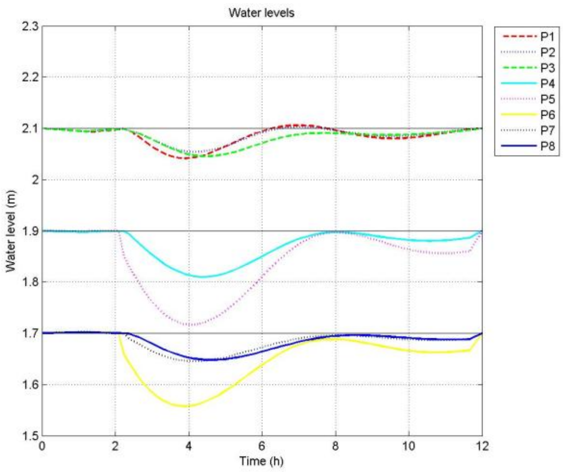

Fig. 5 LQR-Model 1, Water levels

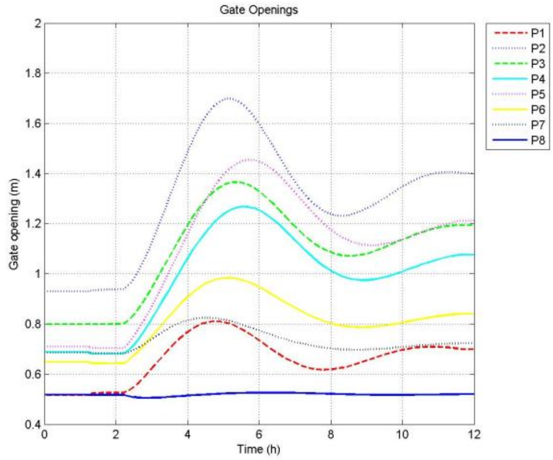

Fig. 6 LQR -Model 1, Gate openings

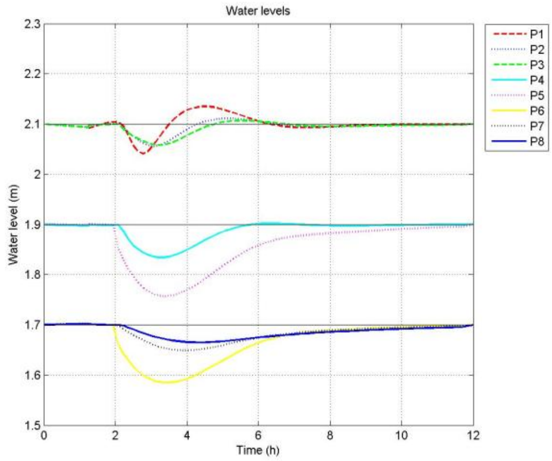

Fig. 7. LQR -Model 2, Water levels

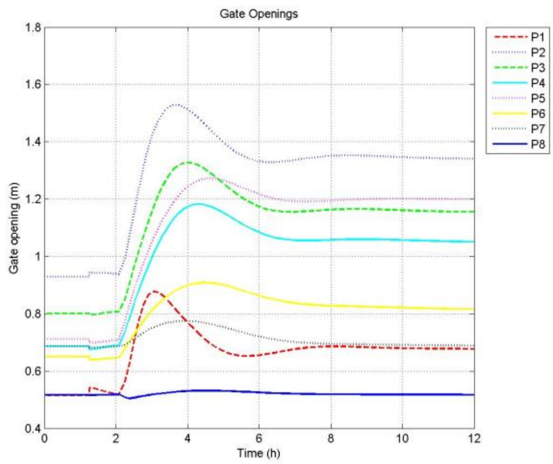

Fig. 8. LQR -Model 2, Gate openings

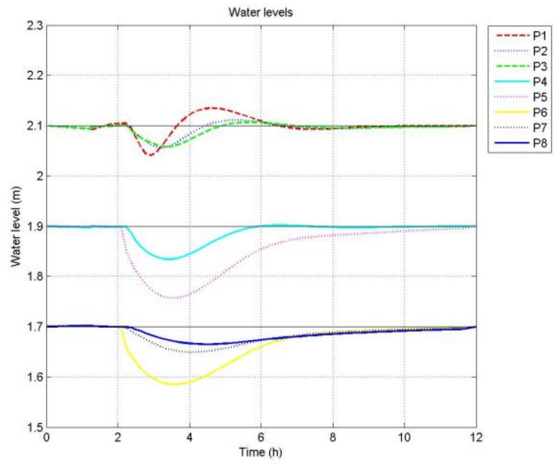

Fig. 9. LQR -Model 3, Water levels

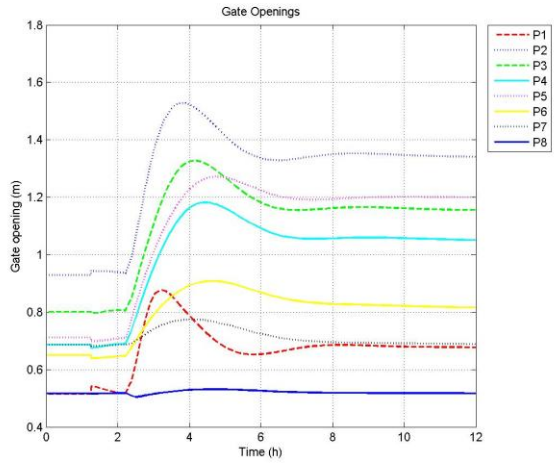

Fig. 10. LQR -Model 3, Gate openings

V. RESULTS AND DISCUSSION

The three models were tested as internal models for LQR centralized controllers using the ASCE Test Canal 2. The results of Model 1 are shown in Fig. 5 and Fig. 6 and the performance indices are shown in Table 1. It can be seen that even with this simple modeling approach a fairly good controller is possible to be developed.

The results with Model 2 are shown in Fig. 8 and Fig. 9, while the performance indices are shown in Table 2. The controller gave a good response. Within 4 hours the water level reaches the setpoint, without offset. The same can be said about the results with Model 3 (Figs. 9-10 and Table 3). These two models showed very similar performance. However, in this case it is important to emphasize that Model 3 had no measurement data about the water levels just downstream the gates (h_2). Hence this is the only model that does not require these water level measurements. In consequence, the implementation of the control system can be more economic. This is a significant advantage of Model 3.

VI. CONCLUSION

Different distant downstream water level control configurations were compared: (1) the discharge as control action variable along with the use of the inverse of the gate equation, and (2) the gate opening as direct control action variable. In the second case there were two possibilities: either measure or model the water level directly downstream of the gates. Simulation results showed better performance in case of the last two models. Model 2 and Model 3 had almost the same performance, considerably better than that of Model 1. These differences are expected to be even greater in case of using model predictive control, where the gate dynamics is considered at every step during the whole prediction horizon.

These results showed that it may be beneficial to use gate opening as control variable and, moreover, it might also be possible to avoid the measurements of the downstream water levels of the gates.

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